Toward unification: the dimensionless equation of motion

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We demonstrate that if masses and charges figuring in the equation of motion including both Newton gravitational and Coulomb electrostatic force laws are divided by mass and charge, respectively, which are derived using the relations contaning only the fundamental physical and mathematical constants (like relations defining the Planck's mass, length, and time), then the gravitational constant and permitivity of vacuum can be eliminated from the equation. In addition, the equation becomes dimensionless containing only the ratios of distances, velocities, masses, and electric charges. The ratios of masses and charges can further be replaced with the ratios of wave-lengths or frequencies. The corresponding equation of motion implies that the fundamental physical constants as the gravitational constant, permitivity of vacuum, and Planck constant are, likely, mere the transformation constants between artificial quantities as mass and electric charge, which were established by man to communicate some concerning things and events in every-day life, and natural physical quantities as wave-length or frequency of oscillations of waving space-time.

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I. INTRODUCTION

It is known that Albert Einstein during a long period of his scientific career attempted to work out a unified theory of all forces in the nature. He expected that such a theory would be completely geometrical. The physical constants should be eliminated or calculated one with the help of others.

The goal attempted by Einstein and many his followers seems to be achieved, at least in part, within the being-newly-born theory of the unification based on the Maxwell electromagnetism[1]. The outline of this theory provides a dimensionless equation of motion of an electrically charged particles in the force field of both gravity and electric force. Unfortunately, the theory is not yet complete. Besides other, the theory has an ambition to include the description of atom within the unified concept, but only a part of energy levels in the hydrogen atom can, at this moment, be determined, those characterized with the quantum numbers $l = n - 1$ and $j = l + 1/2$ in the Dirac's theory. Further, the theory is expected to provide an exact description of the mechanism of interaction, i.e. the absorption of the wave associated with a test particle (TP) by an acting particle (AP). At the randomly distributed phase shift between the oscillations of the TP wave and AP itself, the model should result in relation

$$\frac{<I_\phi>}{2I_t} = 4\pi\alpha, \tag{1}$$

where $<I_\phi>$ is the integral characterizing the mean absorption in unit of the maximum absorption, $I_t = 2/\pi$ is the mean appearance of the TP wave in the place of AP in unit of maximum appearance, and α is the fine structure constant. The preliminary model based on a scalling law gives $<I_\phi>= (1 - 2/\pi)/\pi$, which implies the calculated value of fine structure constant $\alpha_c = (1 - 2/\pi)/(16\pi) = 1/138.33$, i.e. too much (about 1%) differing from the actual experimental value.

It nevertheless appears that the dimensionless equation of motion can also be derived regardless any new theory. In this short research note, we describe this derivation and discuss some implications of the dimensionless equation of motion toward the unification of gravity with the electric interaction.

II. EQUATION DERIVED USING THE PLANCK QUANTITIES

Let us consider a TP in the coordinate frame in which this particle is in rest and an AP forcing the TP to accelerate. The TP is situated in distance r from the AP. The mass and electric charge of the TP (AP) are m_T and q_T (m_A and q_A), respectively. The acceleration of the TP, i.e. the change of its velocity Δv per a time interval Δt, can be calculated from its equation of motion:

$$m_T\frac{\Delta v}{\Delta t} = -G\frac{m_T m_A}{r^2} + \frac{1}{4\pi\varepsilon_o}\frac{q_T q_A}{r^2}, \tag{2}$$

where G is the gravitational constant and ε_o is the permitivity of vacuum.

The dimensional analysis enables combining the fundamental physical constants to obtain special length, time, and mass, which are known as the Planck length, L_P, Planck time, t_P, and Planck mass, M_P. Specifically, these quantities are defined by

$$L_P = \sqrt{\frac{G\hbar}{c^3}},\quad t_P = \frac{L_P}{c} = \sqrt{\frac{G\hbar}{c^5}},\quad M_P = \frac{\hbar}{cL_P} = \sqrt{\frac{\hbar c}{G}} \tag{3}$$

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In these relations, c is the velocity of light and $\hbar$ is the Planck's constant divided by 2π. Further, we know that the fine structure constant can be given as $\alpha = q_o^2/(4\pi\varepsilon_o\hbar c)$ (q_o is the elementary electric charge), from which $1/(4\pi\varepsilon_o) = \alpha\hbar c/q_o^2$. If G is expressed with the help of the Planck's mass, i.e. $G = \hbar c/M_P^2$, and we assume the change of velocity Δv during the Planck's time, i.e. $\Delta t = t_P$, then Eq.(2) can be re-written to form

$$m_T \frac{M_P c^2}{\hbar}\Delta v = -\frac{\hbar c}{r^2}\frac{m_T m_A}{M_P^2} + \frac{\alpha\hbar c}{r^2}\frac{q_T q_A}{q_o^2}. \qquad (4)$$

Further, if we multiply the right-hand side of this equation with $L_P^2 M_o^2 c^2/\hbar^2$ (i.e. with unity) and, then, multiply the whole equation with $\hbar/(M_P c^2)$, the equation acquires the dimensionless form

$$\frac{m_T}{M_P}\frac{\Delta v}{c} = -\left(\frac{L_P}{r}\right)^2 \frac{m_T}{M_P}\frac{m_A}{M_P} + \alpha\left(\frac{L_P}{r}\right)^2 \frac{q_T}{q_o}\frac{q_A}{q_o}. \qquad (5)$$

III. USAGE OF ELEMENTARY ELECTROMASS AND RELATED QUANTITIES

A more natural way to define the fundamental interval of mass and, subsequently, length and time than a pure dimensional analysis is a formal unification of Newton gravitational and Coulomb electrostatic laws. Specifically, we define such a mass, M_o, that two particles having this mass attract each other with the same force as particles charged with two (positive and negative) elementary electric charges. It means, we require the validity of $GM_o^2/r^2 = q_o^2/(4\pi\varepsilon_o r^2)$. The latter yields $M_o = q_o/\sqrt{4\pi\varepsilon_o G} = \sqrt{\alpha}M_P$. We refer to this mass as the "elementary electromass", hereinafter. The interval of length appears to be suitably defined as the Compton wave-length corresponding to M_o divided by the factor of 2π. In the new theory of unification, mentioned in Sect. 1, this length is called as "interaction radius", R_I. According to our definition, $R_I = \hbar/(2\pi M_o c)$. Finally, let us assume that the TP accelerates during the time interval equal to $\Delta t = 2R_I/c = t_P/(\pi\sqrt{\alpha})$. With the new set of fundamental intervals, Eq.(2) can be re-written to the form

$$\frac{m_T}{M_o}\frac{\Delta v}{c} = 4\pi\alpha\left(\frac{R_I}{r}\right)^2\left(-\frac{m_T}{M_o}\frac{m_A}{M_o} + \frac{q_T}{q_o}\frac{q_A}{q_o}\right). \qquad (6)$$

Eqs.(5) and (6) are both dimensionless, because only the dimensionless quantities and the ratios of masses, velocities, lengths, and charges, which are of course also dimensionless, occur.

A pure geometrical description can, in principle, contain only two quatities: length and, if the geometry is evolving in time, the time gradient of lenght. Eqs.(5) as well as (6) can acquire the form containing only these quantities when we assume the TP and AP consisting of only positively and negatively charged elementary particles (EPs). Specifically, we assume that the TP (AP) consists of N_+ (n_+) positively charged EPs, each with mass m_+ and charge $+q_o$, and N_- (n_-) negatively charged EPs with mass m_- and charge $-q_o$. So, $m_T = N_+m_+ + N_-m_-$, $m_A = n_+m_+ + n_-m_-$, $q_T = (N_+ - N_-)q_o$, and $q_A = (n_+ - n_-)q_o$. Using this assumption and denotation, Eq.(6) can be written in form

$$\left(N_+\frac{m_+}{M_o} + N_-\frac{m_-}{M_o}\right)\frac{\Delta v}{c} = 4\pi\alpha\left(\frac{R_I}{r}\right)^2 \cdot$$
$$\cdot\left[-\left(N_+\frac{m_+}{M_o} + N_-\frac{m_-}{M_o}\right)\left(n_+\frac{m_+}{M_o} + n_-\frac{m_-}{M_o}\right) + \right.$$
$$\left. + (N_+ - N_-)(n_+ - n_-)\right]. \qquad (7)$$

Further, we can use the well-know de Broglie's relation between the mass of particle and the frequency of its associated wave, ν, i.e. $mc^2 = h\nu$ (h is the original Planck's constant). If the frequency is converted to the wave-length, λ, according to $\nu = c/\lambda$, then mass can be given as $m = h/(c\lambda)$. We also assign an associated wave, with wave-length Λ_o, to the elementary electromass. Using these conversions and writing corresponding subscripts in the denotation, Eq.(7) can be, after some algebraic handling, written in another form:

$$\left(N_+\frac{\Lambda_o}{\lambda_+} + N_-\frac{\Lambda_o}{\lambda_-}\right)\frac{\Delta v}{c} = 4\pi\alpha\left(\frac{R_I}{r}\right)^2 \cdot$$
$$\cdot\left[-\left(N_+\frac{\Lambda_o}{\lambda_+} + N_-\frac{\Lambda_o}{\lambda_-}\right)\left(n_+\frac{\lambda_o}{\lambda_+} + n_-\frac{\Lambda_o}{\lambda_-}\right) + \right.$$
$$\left. + (N_+ - N_-)(n_+ - n_-)\right]. \qquad (8)$$

Ratio $\Delta v/c$ in Eq.(8) expresses a gradient of change of the TP position given in unit of change of position of a photon moving with the velocity of light. Besides the "gradient" of change of the EP position (length), Eq.(8) contains only ratios of length and numbers of EPs. So, it satisfies the requirements of pure geometrical description. The masses and charges can be eliminated in an analogous way also from Eq.(5).

One could, perhaps, object that the fine structure constant, figuring in Eq.(8), though dimensionless is still physical constant and, therefore, this equation is not purely geometrical. This problem can be removed assuming an alternative representation of α. In Sect. 1, we introduced an example, where α was related, by Eq.(1), to geometrical aspects of the mechanism of interaction. Accepting, e.g., this representation, α could be calculated from the geometry of interaction and was not, thus, the true physical, but rather geometrical (i.e. mathematical) constant.

IV. SOME STEPS TOWARD THE UNIFICATION

In the previous part, we considered the wave associated with particle, which uses to be described by wave equation. The amplitude of the wave, in one-dimensional

case, is often obtained in form of exponential function $\exp(\pm ikr)$, where i is the unit of imaginary numbers and k is the size of wave-vector. If interaction is assumed to be mediated by its associated wave and signal is received by the particle at an extremely short distance ($r = R_I$), then the argument $\pm ikr \rightarrow \pm ikR_I$. In a simple case, the size of wave-vector can be given as the ratio of angular frequency, ω, and velocity of light, and the former can be given with the help of mass, m, according the well-known de Broglie's relation $mc^2 = \hbar\omega$. So, the argument $\pm ikR_I$ can be gauged to become $\pm im/M_o$. We note that $m/M_o \ll 1$ for all EPs, therefore the exponential can be approximted, with a high precision, as $\exp(\pm im/M_o) = 1 \pm im/M_o$. This amplitude can further be gauged as $-i(1 + im_+/M_o)$ when corresponding with the positively charged EP and as $+i(1 - im_-/M_o)$, i.e. as complex-number conjugate, when corresponding to the negatively charged EP.

In electromagnetism as well as quantum physics, the state of a system sometimes uses to be described by the complex-valued functions, the real parts of which correspond to the actually observed effects. If we assume that the force between two (point-like) objects is proportional to the appropriate sums of the above-mentioned amplitudes, which equal to $-iN_+(1 + im_+/M_o) + iN_-(1 - im_-/M_o)$ for a test object and $-in_+(1 + im_+/M_o) + in_-(1 - im_-/M_o)$ for an acting object, then it is possible to prove that the real part of the product of multiplication of these two sums is identical to the form in brackets of Eq.(7).

Interestingly, the form in the brackets in Eq.(7) includes not only the Coulomb electrostatic, but also the Newton gravitational law and we just reproduced these brackets only with the amplitudes of functions which can occur in the solution of Maxwell equations. The gravity really seems to be comprehended by the theory of electromagnetism. In a neutral body, for which $N_+ = N_- \equiv N$, sum $-iN(1 + im_+/M_o) + iN(1 - im_-/M_o)$ is equal to $N(m_+ + m_-)$. It means, the first terms ($-iN$ and $+iN$) of the exponentials, expanded to power series of argument, vanish, but the second terms (Nm_+/M_o and Nm_-/M_o) survive. And, notice that the result $N(m_+ + m_-)$ is, in fact, the mass of the neutral body consisting of N positively charged EPs, with mass m_+, and N negatively charged EPs, with mass m_-.

V. CONSEQUENT REPRESENTATION OF PHYSICAL CONSTANTS

In Sects. 2 and 3, we could see that if masses and charges figuring in the equation of motion including both Newton gravitational and Coulomb electrostatic force laws are divided by mass and charge, respectively, which are derived using the relations contaning only the fundamental physical and mathematical constants, then the gravitational constant and permitivity of vacuum can be eliminated from the equation.

The true meaning of the constants G, ϵ_o, and h (or $\hbar$) seems to be indicated by Eq.(8) in which also the masses and charges are eliminated. In this equation, we meet only the distances and wave-lengths (which could be converted, eventually, to frequencies), besides the gradient of the change of position. Seeing this equation, it seems that the nature "knows" only the curved and oscillating space-time and change in position of wave sources.

Actually, the concept of mass evolved from the concept of weight of material bodies in the Earth's gravity and concept of electric charge was established as a certain analogy to the concept of mass. The weight/mass was not initially any physical quantity, but people established this concept in dawn of ages to mutually communicate, also quantitatively, the measure of some goods or effects of the Earth's attraction of material things. When the force laws were later created using the concepts of mass and/or charge, there had to be established some "transformation constants" between the artificially defined human quantities (mass, charge) and natural physical quantities (wave-lengths or frequencies of oscillations). If human quantities are not used in the formulation of force laws, no transformation constants, as seen in Eq.(8), are needed. Just this seems to be the role of such the fundamental physical constants as gravitational constant, permitivity of vacuum, and the Planck constant.

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[1] arXiv:1012.5763v1 [physics.gen-ph] 28 Dec 2010